1. **General Definition :** If to every value (Considered as real unless other-wise stated) of a variable x, which belongs to some collection (Set) A, there corresponds one and only one finite value of the quantity y, then y is said to be a function (Single valued) of x or a dependent variable defined on the set A; x is the argument or independent variable.

If to every value of x belonging to some set A there corresponds one or several values of the variable y, then y is called a multiple valued function of x defined on A. Conventionally the word "**Function**" is used only as the meaning of a single valued function, if not otherwise stated. Pictorially: $\xrightarrow[input]{x} \boxed{f} \xrightarrow[output]{f(x)=y}$, y is called the image of x and x is the pre-image of y under f.
Every function from $A \to B$ satisfies the following conditions.
a) $f \subset A \times B$
b) $\forall a \in A \Rightarrow (a,\ f(a)) \in f$ and
c) $(a,\ b) \in f\ \&\ (a,\ c) \in f\ \Rightarrow b = c$

2. **Domain, Co-Domain & Range Of a Function :** Let $f : A \to B$, then the set A is known as the domain of f and the set B is known as co-domain of f. The set of all "f" images of elements of A is known as the range of f. Thus :
Domain of $f = \{a \mid a \in A,\ (a,\ f(a)) \in f\}$
Range of $f = \{f(a) \mid a \in A,\ f(a) \in B\}$

It should be noted that range is a subset of co-domain. Sometimes if only f(x) is given then domain is set of those values of "x" for which f(x) exists or is defined.
To find the range of a function, there is n"t any particular approach, but student will find one of these approaches useful.
i) When a function is given in the form y = f(x), express if possible "x" as a function of "y" i.e. x = g(y). Find the domain of "g". This will become range of "f".
ii) If y = f(x) is a continuous or piece-wise continuous function, then range of "f" will be union of $[\text{Min}^{m} f(x), \text{Max}^{m} f(x)]$ in all such intervals where f(x) is continuous/piece-wise continuous.

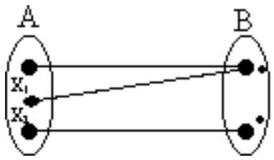
3. **Classification of Functions :**
Functions can be classified into two categories :
i) **One-One Function (Injective mapping) or Many - One Function :** A function is said to be a one-one function or injective mapping if different elements of A have different f images in B. Thus for .

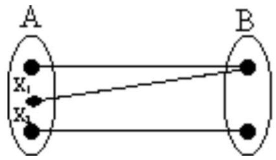
$x_1,\ x_2 \in A \,\&\, f(x_1),\ f(x_2) \in B,\ f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2 \ or\ x_1 \neq x_2 \Leftrightarrow f(x_1) \neq f(x_2)$
Diagramatically an injective mapping can be shown as

OR
Note : (a) Any function which is entirely increasing or decreasing in whole domain, then f(x) is one-one.
(b) If any line parallel to x-axis cuts the graph of the function atmost at one point, then the function is one-one.
Many - One Function : A function $f : A \to B$ is said to be a many one function if two or more elements of A have the same f image in B. Thus $f : A \to B$ is many one if for :

$x_1, x_2 \in A,\ f(x_1) = f(x_2)$ but $x_1 \neq x_2$

Diagramatically a many one mapping can be shown as



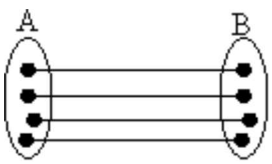


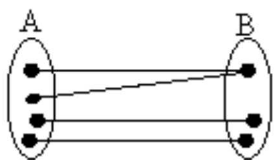
OR

Note : (a) Any continuous function which has atleast one local maximum or local minimum, then f(x) is many-one. In other words, if there is even a single line parallel to x- axis cuts he graph of the function atleast at two points, then f is many - one.
(b) If a function is one-one, it cannot be many-one and vice versa.
(c) All functions can be categorized as one-one or many-one

ii) Onto function (Surjective mapping) or into function :
If the function $f : A \rightarrow B$ is such that each element in B (co-domain) must have atleast one pre-image in A, then we say that f is a function of A "onto" B. Thus $f : A \rightarrow B$ is surjective iff $\forall\ b \in B$, $\exists$some $a \in A$ such that f(a) = b.

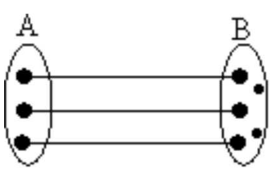
Diagramatically surjective mapping can be shown as

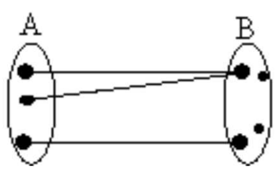




OR
Note that : If range = Co-domain, then f(x) is onto.

Into Function : If $f : A \rightarrow B$ is such that there exists atleast one element in co-domain which is not the image of any element in domain, then f(x) is into.
Diagramatically into function can be shown as





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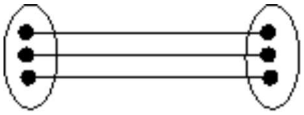

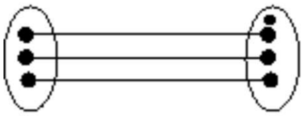

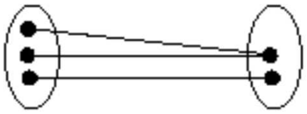

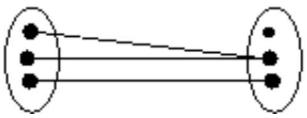

Note that : If a function is onto, it cannot be into and vice versa.
Thus a function can be one of these four types :
a) one-one onto (injective and surjective)
b) one-one into (injective and surjective)
c) many - one onto (surjective but not injective)
d) many-one into (neither surjective nor injective)
(domain in each case is $A\cap B$)

Note : a) If f is both injective and surjective, then it is called a **Bijective** mapping. The bijective functions are also named as invertible, non-singular or biuniform functions.

b) If a set A contains n distinct elements then the number of different functions defined from $A \to A$ is n^n and out of it n! are one one.